Toward a statistical characterization of the Galactic foregrounds on the full sky.

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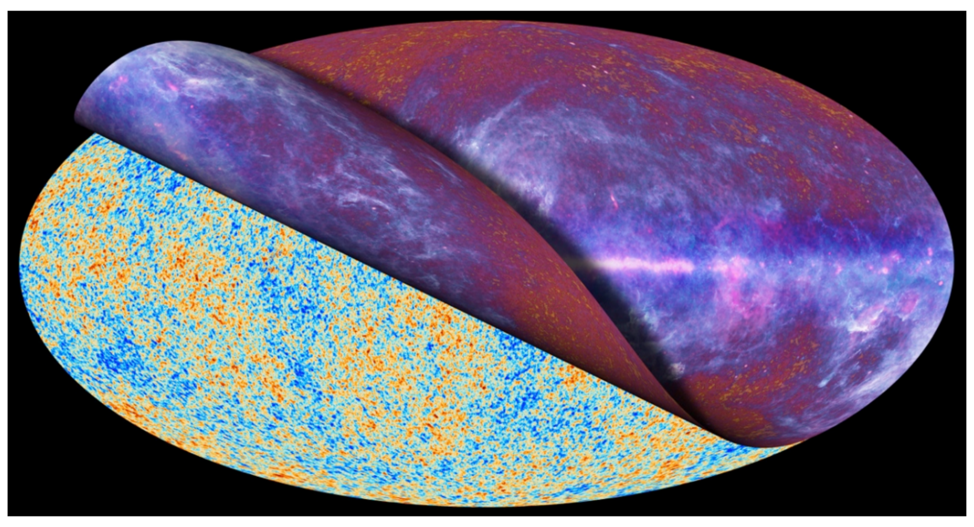
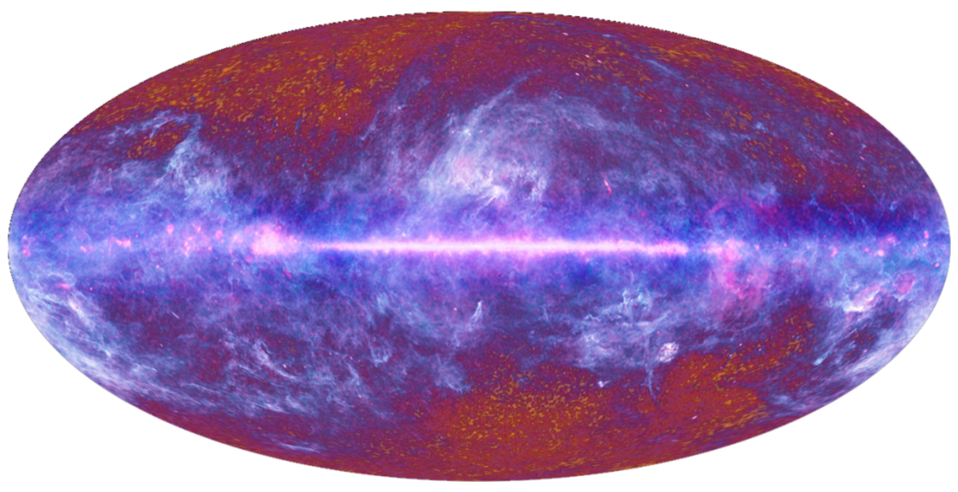
Abstract:

Galactic foreground removal is a main issue for CMB data analysis and the detection of primordial B-mode polarization. The interstellar medium evolves under a large number of physical processes such as gravity, radiation or magnetohydrodynamics. Because highly non-linear physics is involved, the Galactic dust spatial structuration is non-gaussian, showing filamentary structures. This characteristic is a potential lever arm to disentangle it from the CMB Gaussian field. One possibility to statistically characterize the non-gaussianities is to use Scattering Transforms [1]. These statistics are inspired from neural networks but can be computed from a single image, without a training step. They have already been applied in the field of astrophysics [2, 3]: estimation of physical parameters, production of synthetic maps from a single observation that present the same statistics, separation of Galactic dust and instrumental noise. These statistics were originally developed for planar maps. Their extension to a spherical geometry is an important step toward the analysis of large sky survey data. This is especially required for the LiteBIRD space mission that will target primordial B-modes which are expected at very large angular scales.



Galactic foregrounds are highly non Gaussian:



Non linear physical processes: gravity, radiation, magneto-hydrodynamic... → Coupling between spatial scales → Non Gaussian spatial structures (ex: filaments)

We need statistics to describe the couplings between scales.

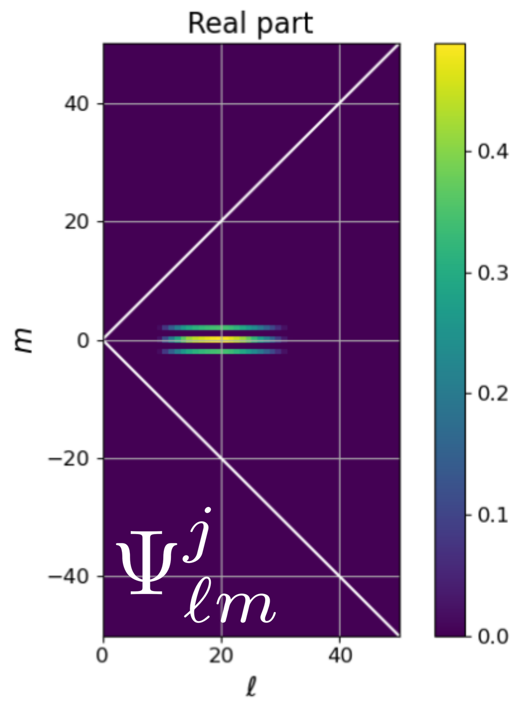
Scattering Transforms (ST):

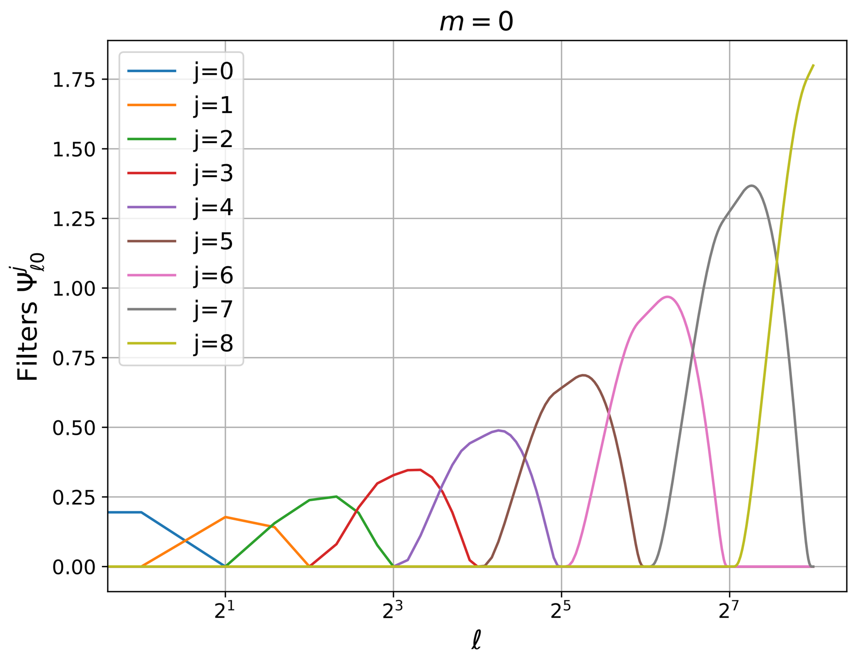
- A set of statistical coefficients to describe the non Gaussianity of a signal.
- Inspired from neural networks but can be written explicitely.
- Based on the convolution of the signal by a filter set with different scales and orientations.

Filter set on the sphere:

Filters are defined in the spherical harmonic space.

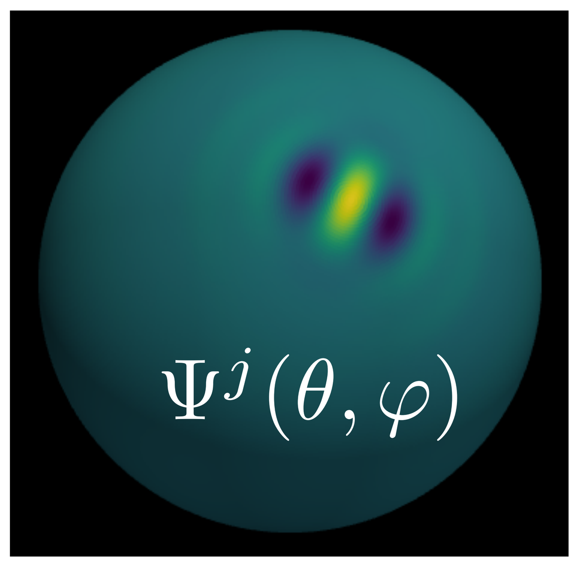
j = filter scale.



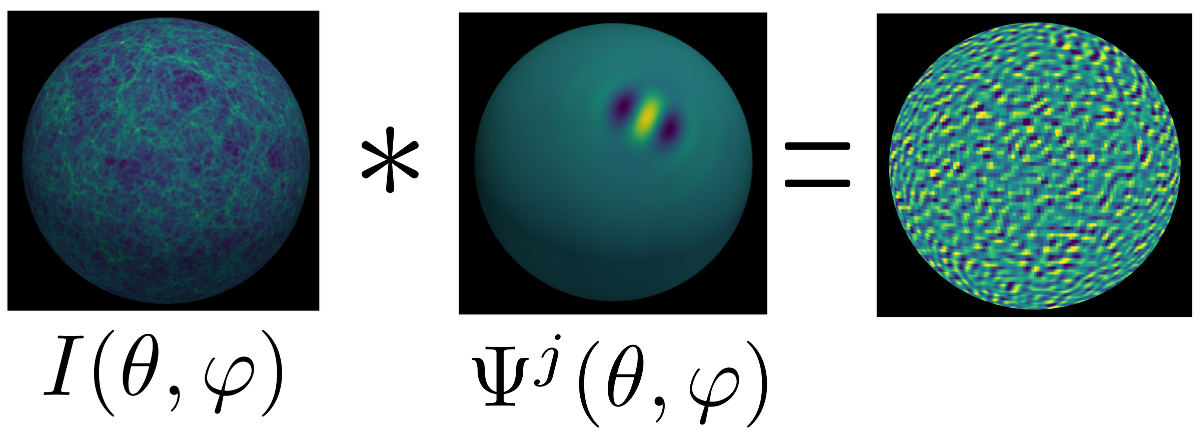


Spherical harmonic transform:

$$\Psi(\theta,\varphi) = \sum_{\ell m} \Psi_{\ell m} Y_{\ell m}(\theta,\varphi)$$



Directional convolution [4]:



Wavelet transform:

$$W^j_{\ell m n} = \frac{8\pi^2}{2\ell+1} I_{\ell m} \Psi^{j*}_{\ell n}$$

↓ Inverse Wigner Transform

$$W^j_\gamma(\theta,\varphi)$$

Convolved map with orientation γ

Scattering covariance coefficients [5]:

L1 and L2 norms:

$$S^j_1 = \langle |I * \Psi^j| \rangle$$

$$P^j_{00} = \langle |I * \Psi^j|^2 \rangle$$

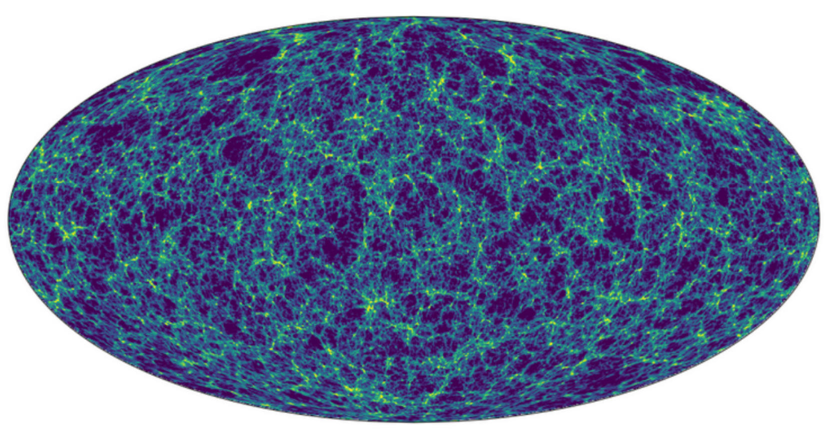
Tipically around 6 to 9 scales and 3 to 5 angles.
=> about few thousand of coefficients.

Covariances between scales:

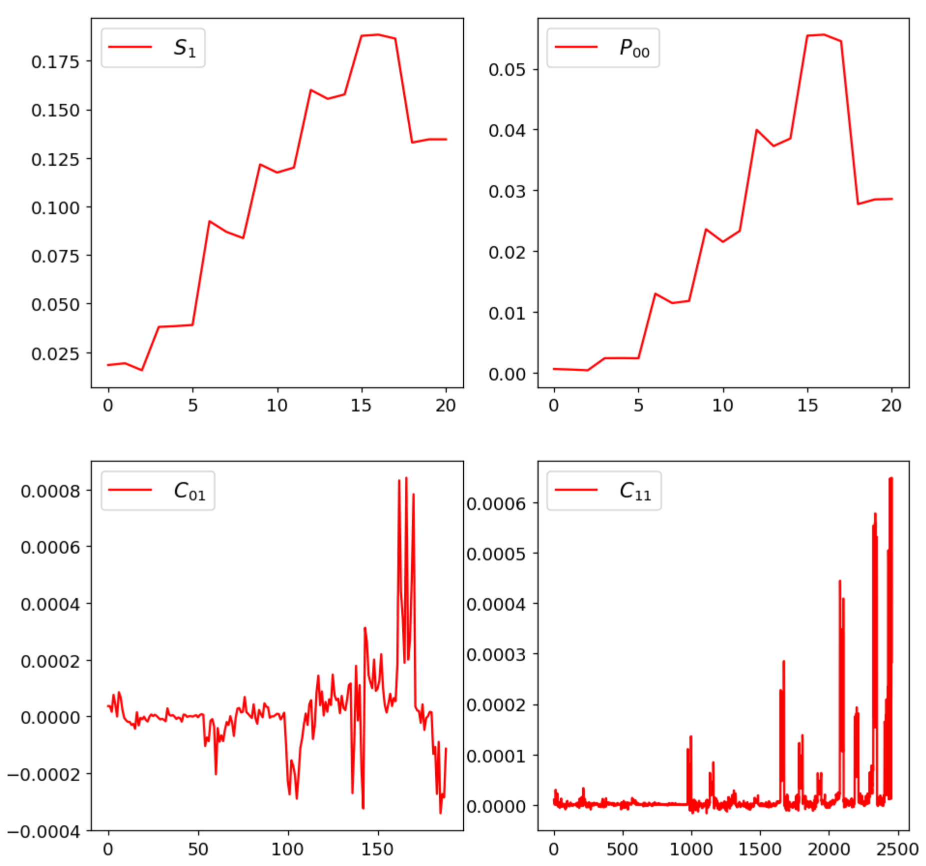
$$C^{j_1 j_2}_{01} = \mathrm{Cov}\left[I * \Psi^{j_2}, |I * \Psi^{j_1}| * \Psi^{j_2}\right]$$

$$C^{j_1 j_2 j_3}_{11} = \mathrm{Cov}\left[|I * \Psi^{j_1}| * \Psi^{j_3}, |I * \Psi^{j_2}| * \Psi^{j_3}\right]$$

The coefficients can be computed from a single observation.



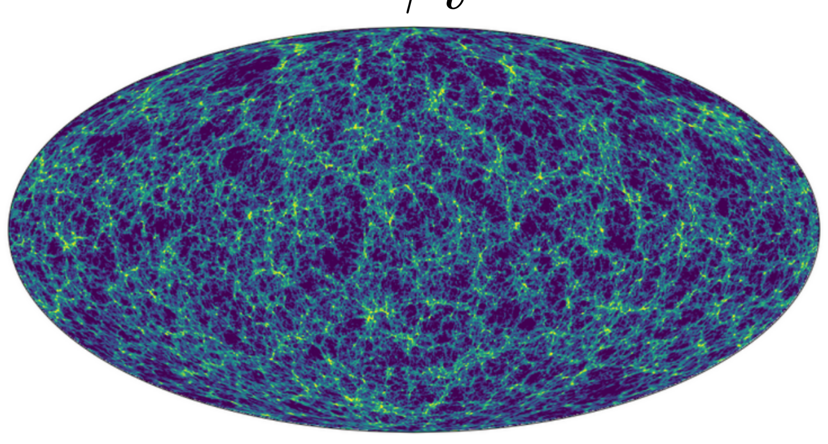
LSS N-body simulation [6]



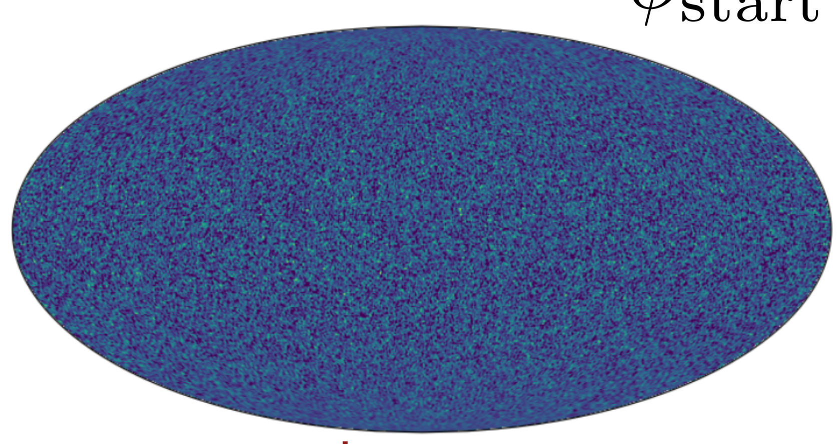
Validation on Large Scale Structure (LSS) maps :

Maximum entropy generative model: from the scattering covariance coefficients of the target field, generate a new field which has similar structures and statistical properties.

Target field with scattering coefficients ϕ_t



Initial condition: white noise ϕ_{start}



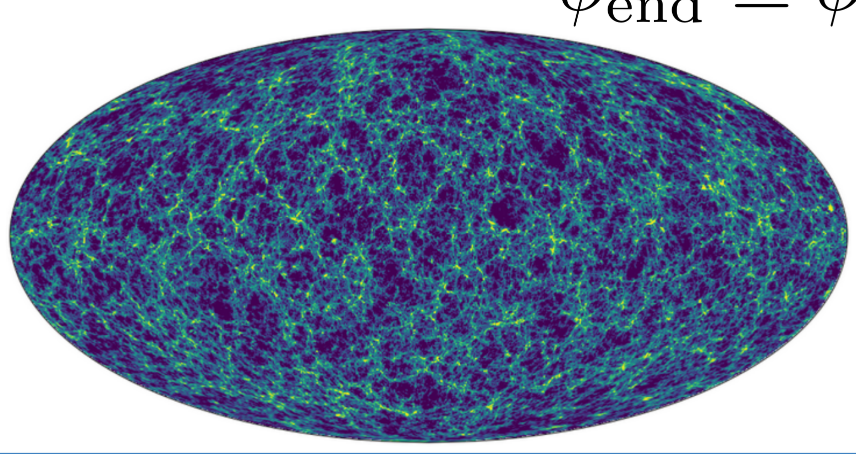
↓ Gradient descent
Automatic differentiacion on GPU

Loss function to be minimize iteratively varying the pixel values of the initial map or its spherical harmonic coefficients:

$$\mathcal{L} = |\phi_t - \phi_i|^2$$

↑ Scattering coefficients at iteration i

Synthetized field $\phi_{\text{end}} \simeq \phi_t$



References:

[1] J. Bruna et al., "Invariant Scattering Convolution Networks", 2013
[2] E. Allys et al., "The RWST, a comprehensive statistical description of the non-Gaussian structures in the ISM", 2019
[3] B. Regaldo-Saint Blancard et al., "Statistical description of dust polarized emission from the diffuse interstellar medium - A RWST approach", 2020
[4] J. D. McEwen et al., "Directional Spin Wavelets on the Sphere", 2015
[5] R. Morel et al., "Scale Dependencies and Self-Similarity Through Wavelet Scattering Covariance", 2022
[6] T. Kacprzak et al., "CosmoGridV1 : A Simulated wCDM Theory Prediction for Map-Level Cosmological Inference", 2023

Perspectives:

- Publication of this method, Mousset et al. 2023 [in prep]
- Adapt it to Galactic foreground synthesis
- Development of a component separation algorithme
- Cosmic string map synthesis, Price et al. 2023 [in prep]